Self-organized criticality in ^{4}He with a heat current

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In the presence of a small heat current and appropriate boundary conditions, ^{4}He will reach the superfluid critical point without fine tuning the temperature. This provides a physical example of self-organized criticality via the mechanism of singular diffusion proposed by Carlson *et al.* [Phys. Rev. Lett. **65**, 2547 (1990)].

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Most equilibrium systems require the fine tuning of parameters to be held at a critical point; however, open systems may be driven to a critical point under a range of conditions. The phenomena of self-organized criticality [1] has aroused considerable theoretical interest but experimental realizations have been lacking. Carlson *et al.* [2] showed that systems which possess a singular diffusion coefficient at a critical point will display self-organized criticality. In this note we point out that the singularity in thermal conductivity at the λ point in ^{4}He leads to self-organized criticality via the mechanism discussed in [2].

A self-organized λ point may be obtained in a large sample of liquid ^{4}He. The system is thermally insulated except for two open faces separated by a distance L. One face is held at a temperature $T_0 > T_\lambda$, while a fixed heat flux $\dot{Q}$ is maintained across the other face. Because the thermal conductivity of liquid ^{4}He diverges at the λ point, the ideas of Ref. [2] are applicable. Near the hot end of the sample there is a boundary layer where the temperature gradient is nonzero but elsewhere the temperature will be very near T_λ. For small $\dot{Q}$ the temperature in the sample cannot fall much below T_λ because of the infinite thermal conductivity of the superfluid state. On the other hand, if the distance L between the hot and cold faces is large, a given $T_0 > T_\lambda$ and $\dot{Q} > 0$ are possible only if the thermal conductivity is very large throughout most of the length of the sample. The critical point is self-organized since T_0 may be set within a wide range affecting only the boundary layer. The same ideas can be applied in a two-dimensional geometry. If heat is extracted from one side of a ^{4}He film while the other side is maintained at a temperature above the Kosterlitz-Thouless transition temperature then most of the film will remain very near the transition temperature. Note that *mixed* boundary conditions are required to maintain the critical steady state, the entropy current is fixed on one boundary while the conjugate thermodynamic variable, temperature, is fixed on the other.

Because of the presence of nonlinear dissipative mechanisms the superfluid will be maintained slightly below T_λ if $\dot{Q} > 0$, thus $\dot{Q}$ determines deviations from criticality [3,4]. In order to approach criticality one requires $\dot{Q}L \to \infty$, but $\dot{Q} \to 0$. A small-flux requirement is characteristic of all self-organized critical systems.

The fact that helium may be maintained near the λ point via a heat current is familiar to low-temperature experimentalists and is exploited in a commercially available device for cooling superconducting magnets, the "λ-point refrigerator" [5]. In this device, a cooling coil in a tall cylinder of liquid helium creates a pool in the bottom of the cylinder consisting of superfluid just below the λ point. The top surface of the bath is in contact with vapor at atmospheric pressure and $T = 4.2$ K.

While much is known about the region near the superfluid critical point in the presence of a heat current [3,4,6,7] it would be interesting to reexamine this system from the point of view of self-organized criticality. For example, cellular-automata models of self-organized criticality display "avalanches" and power-law distributions of current fluctuations [1]; do analogous phenomena occur at the self-organized superfluid transition?

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